Aufgabe 3:

a)

$$\Phi(R,\vartheta,\varphi) = \frac{\Phi_0}{2}\left(3\vec{e}_z\cdot\vec{R}-1\right) = \Phi_0 P_2(\cos\vartheta)$$

$$\Phi^{a/i} = \sum_{l=0}^{\infty}(2l+1)\left[A_l^{a/i}r^l + B_l^{a/i}r^{-(l+1)}\right]P_l(\cos\vartheta)$$

Außen:

$$\Phi(r\to\infty) = 0 \quad , \quad \Phi^a(R) = \Phi$$

$$\Rightarrow A_l^a = 0 \quad , \quad \Phi^a(R) = \sum_{l=0}^{\infty}(2l+1)b_l^a R^{-(l+1)}P_l(\cos\vartheta) \stackrel{!}{=} \Phi(R) = \Phi_0 P_2(\cos\vartheta)$$

$$\Rightarrow B_l^a = \delta_{l2}\Phi_0\frac{R^3}{5}$$

$$\Phi^i(r\to\infty) \neq \pm\infty \quad \Rightarrow A_l^i = 0$$

$$\Phi^i(R) = \sum_i (2l+1)A_l^i P_l(\cos\vartheta) \stackrel{!}{=} \Phi(R) = \Phi_0 P_2(\cos\vartheta)$$

$$\Rightarrow A_l^i = \delta_{2l}\Phi_0\frac{1}{5R^2}$$

$$\Rightarrow \Phi = \begin{cases}\Phi_0\left(\frac{R}{r}\right)^3 P_2(\cos\vartheta) & \text{außen}\\ \Phi_0\left(\frac{R}{r}\right)^2 P_2(\cos\vartheta) & \text{innen}\end{cases}$$

b)

$$\Rightarrow \Phi_0 = -\frac{2q}{R}$$

c)

$$\left.\frac{\partial\Phi^a}{\partial r}\right|_{r=R} - \left.\frac{\partial\Phi^i}{\partial r}\right|_{r=R} = -4\pi\sigma(\vartheta)$$

$$\sigma(\vartheta) = \frac{5}{4\pi}\frac{\Phi_0}{R}P_l(\cos\vartheta)$$