Aufgabe 1:

a)

$$A'^{\mu} = \Lambda^{\mu}{}_{\nu} A^{\nu}$$

$$A'^{\mu}(x') = \Lambda^{\mu}{}_{\nu} A^{\nu}(x(x'))$$

System (II):

$$A^{\mu}_{II} = \left(\frac{q}{r_{II}}, \vec{0}\right) \quad , \quad r^2_{II} = x^2_{II} + y^2_{II} + z^2_{II}$$

$$\Lambda(I \to II) = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \Rightarrow (x^{\mu}_{II}) = \gamma \begin{pmatrix} ct_I - \beta x_I \\ x_I - \beta c t_I \\ y_I \,/\, \gamma \\ z_I \,/\, \gamma \end{pmatrix}$$

$$A^{\mu}_{I}(x_I) = \Lambda^{\mu}{}_{\nu}(II \to I) A^{\nu}_{II}(x_{II}(x_I)) = \gamma \frac{q}{r_{II}} \begin{pmatrix} 1 \\ \beta \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \gamma \frac{q}{\sqrt{\gamma^2(x_I - \beta c t_I)^2 + y_I^2 + z_I^2}} \\ \gamma\beta \frac{q}{\sqrt{\gamma^2(x_I - \beta c t_I)^2 + y_I^2 + z_I^2}} \\ 0 \\ 0 \end{pmatrix}$$

$$B_I = \mathrm{rot}_I \, \vec{A}_I = \vec{\nabla}_I \times \vec{A}_I \quad , \quad \vec{A}_I = \frac{\gamma q}{r_{II}(x_I)} \begin{pmatrix} \beta \\ 0 \\ 0 \end{pmatrix}$$

$$B_I = \gamma q \begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} \times \frac{1}{r} \begin{pmatrix} \beta \\ 0 \\ 0 \end{pmatrix} = \gamma q \begin{pmatrix} 0 \\ \partial_z \beta \frac{1}{r} \\ -\partial_y \beta \frac{1}{r} \end{pmatrix} = \frac{\gamma q \beta}{r^3_{II}(x_I)} \begin{pmatrix} 0 \\ -z_I \\ y_I \end{pmatrix}$$

$$\vec{E}_I = -\vec{\nabla}_I \Phi_I - \frac{1}{c} \dot{\vec{A}}_I = -\begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} \Phi - \frac{1}{c} \frac{\mathrm{d}}{\mathrm{d}t} \vec{A}_I = \frac{\gamma q}{r^3_{II}} \begin{pmatrix} \gamma^2 (x_I - \beta c t_I) \\ y \\ z \end{pmatrix} - \frac{1}{c} \frac{\beta \gamma q}{r^3} \begin{pmatrix} \gamma^2 (x_I - \beta c t_I)(-\beta c) \\ 0 \\ 0 \end{pmatrix}$$

$$= \frac{\gamma q}{r^3} \begin{pmatrix} \gamma^2 (x + \beta c t - \beta^2 (x + \beta c t)) \\ y \\ z \end{pmatrix} = \frac{\gamma q}{r^3} \begin{pmatrix} x - \beta c t \\ y \\ z \end{pmatrix} \quad \Leftarrow \gamma^2 (1 - \beta^2) = 1$$

In Punkt $P = (0, b, 0)$

$$E_1 = E'_1 = -\frac{q \gamma v t}{(b^2 + \gamma^2 v^2 t^2)^{3/2}}$$

$$E_2 = \gamma E'_2 = \frac{\gamma q b}{(b^2 + \gamma^2 v^2 t^2)^{3/2}}$$

$$B_3 = \gamma \beta E'_2 = \beta E_2$$

Beobachtungen:

$$B_3 = \beta E_2 \quad , \quad \beta \to 1 \quad \Rightarrow B_3 = E_2$$

$$E_2(t = 0) = \frac{\gamma q}{b^2} = \gamma E_0 \quad , \quad |E_2| \sim \gamma$$

$$\frac{1}{b^2 + \gamma^2 v^2 t^2} \sim \frac{1}{1 + \left(\frac{\gamma v t}{b}\right)^2} \quad , \quad \frac{1}{t_0} = \frac{\gamma v}{b} \Rightarrow t_0 = \frac{b}{\gamma v}$$

t_0 ist die charakteristische Zeit, d. h. der Zeitraum, in dem der Term von 0 wesentlich verschieden ist.

b)

$$F_{\mu\nu}F^{\mu\nu} \propto |E|^2 - |B|^2$$
$$B = 0 \quad \Rightarrow F^{\mu\nu}F_{\mu\nu} \geq 0 \quad \Rightarrow |E| \geq |B|$$

c) Simpel!

d) s.o.

Aufgabe 2:

a)

$$\Lambda(I \rightarrow II) = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$R^{II}_{A,n} = \Lambda R^{I}_{A,n}$$
$$\Delta E^{II}_{A} = R^{II}_{A,n} - R^{II}_{A,n-1} = \Lambda(R^{I}_{A,n} - R^{I}_{A,n-1}) = \Lambda \begin{pmatrix} c(t_n - t_{n-1}) \\ nd - (n-1)d \\ 0 \\ 0 \end{pmatrix}$$
$$= \gamma \begin{pmatrix} c(t_n - t_{n-1}) - \beta d \\ d - \beta c(t_n - t_{n-1}) \\ 0 \\ 0 \end{pmatrix} =: \begin{pmatrix} 0 \\ d^{II}_A \\ 0 \\ 0 \end{pmatrix} \quad \text{mit } c(t_n - t_{n-1}) = \beta d$$
$$d^{II}_A = \gamma(d = \beta c(t_n - t_{n-1}) = \gamma g(1 - \beta^2) = \frac{d}{\gamma} < d$$
$$R^{II}_{e,n} = \Lambda R^{I}_{e,n} = \begin{pmatrix} \frac{ct_n}{\gamma} - \beta\gamma nd \\ \gamma nd \\ 0 \\ 0 \end{pmatrix} \quad \Rightarrow \Delta R^{II}_e = R^{II}_{e,n} - R^{II}_{e,n} = \begin{pmatrix} \frac{c}{\gamma}(t_n - t_{n-1}) - \beta\gamma d \\ \gamma d \\ 0 \\ 0 \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} 0 \\ d^{II}_e \\ 0 \\ 0 \end{pmatrix}$$
$$\Rightarrow d^{II}_e = \gamma d > d \Rightarrow d^{II}_e > d^{II}_A$$

b)

$$d^{II} = \gamma d$$
$$B^{II} = 0 \vee \vec{E}^{II} = -\sum_n \frac{e}{\left((x^{II} - n\gamma d)^2 + y^{II^2} + z^{II^2}\right)^{3/2}} \begin{pmatrix} x^{II} - n\gamma d \\ y^{II} \\ z^{II} \end{pmatrix}$$
$$E^{I}_{\parallel} = E^{I}_{x} = E^{II}_{\parallel}$$
$$B^{I}_{\parallel} = B^{I}_{x} = B^{II}_{\parallel} = 0$$
$$E^{I}_{\perp} = \gamma E^{II}_{\perp}$$
$$B^{I}_{\perp} = \vec{\beta}\gamma \times E^{II}_{\perp} = \gamma\beta \begin{pmatrix} 0 \\ -E^{II}_z \\ E^{II}_y \end{pmatrix}$$

Koordinatentrafo:

$$x^{II} = \gamma(x^I - \beta c t^I) \quad , \quad y^I = y^{II} \quad , \quad z^I = z^{II}$$

$$r_n^I = \left[\gamma^2(x^I - \beta c t^I - nd)^2 + {y^I}^2 + {z^I}^2\right]^{1/2}$$

$$\vec{E}^I = -\sum_n \frac{e\gamma}{r_n^3} \begin{pmatrix} x^I - \beta c t^I - nd \\ y^I \\ z^I \end{pmatrix}$$

$$\vec{B}^I = -\sum_n \frac{e\beta\gamma}{r_n^3} \begin{pmatrix} 0 \\ -z^I \\ y^I \end{pmatrix}$$